2/6/18

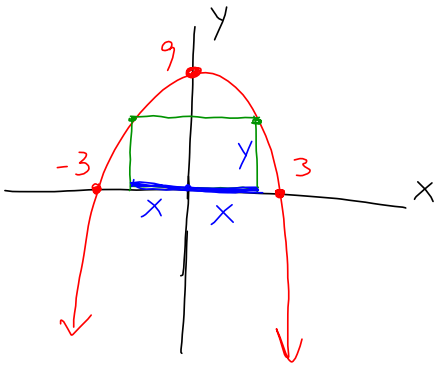
"In order to understand the value in a shortcut, one must have taken the long road first."
-Anonymous

HW: "Optimization Packet" page 123 #1
Test 1 on Thursday 2/15

AIM: Optimization continued

Warm Up:

1. Find the area of the largest rectangle with sides parallel to the axes, whose upper vertices are on $y = 9 - x^2$ and whose lower vertices are on the x-axis.



$0 \le x \le 3$

$0 \le y \le 9$

Area $= 2xy$

$y = 9 - x^2$

$A = 2x(9 - x^2)$

$A = 18x - 2x^3$

$A' = 18 - 6x^2$

$0 = 18 - 6x^2$

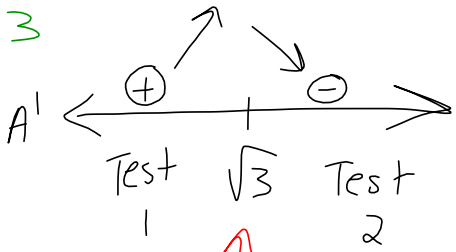
$\frac{6x^2}{6} = \frac{18}{6}$

$x^2 = 3$

$x = \pm\sqrt{3}$

reject $-\sqrt{3}$ b/c of restrictions

$x = \sqrt{3}$



max

$x = \sqrt{3}$

Find y:

$y = 9 - x^2$

$y = 9 - \sqrt{3}^2$

$y = 6$

Area $= 2\sqrt{3} \cdot 6$

Max Area $= 12\sqrt{3}$ units^2

Optimization Problems - Homework

1. Find two numbers whose sum is 10 for which the sum of their squares is a minimum.

$$S = x^2 + (10 - x)^2$$
$$S = 2x^2 - 20x + 100$$
$$4x - 20 = 0$$
$$x = 5, y = 5$$

2. Find nonnegative numbers x and y whose sum is 75 and for which the value of xy^2 is as large as possible.

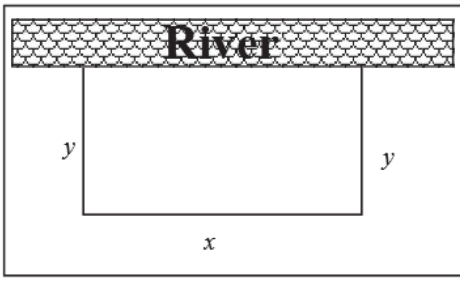
$$P = (75 - y)y^2$$
$$P = 75y^2 - y^3$$
$$0 = 150y - 3y^2$$

$$3y(50 - y) = 0$$
$$y = 50, x = 25$$

3. A ball is thrown straight up in the air from ground level. Its height after t seconds is given by $s(t) = -16t^2 + 50t$. When does the ball reach it maximum height? What is its maximum height?

$$0 = -32t + 50$$
$$32t = 50$$
$$t = \frac{25}{16} \text{ sec}, \; s(t) = 39.063 \text{ ft}$$

4. A farmer has 2,000 feet of fencing to enclose a pasture area. The field will be in the shape of a rectangle and will be placed against a river where there is no fencing needed. What is the largest area field that can be created and what are its dimensions?



$$P = x + 2y = 2000$$
$$A = xy$$
$$A = (2000 - 2y)y$$
$$A = 2000y - 2y^2$$
$$0 = 2000 - 4y$$
$$4y = 2000$$
$$y = 500 \text{ ft}, x = 1000 \text{ ft}, \text{Area} = 500{,}000 \text{ ft}^2$$

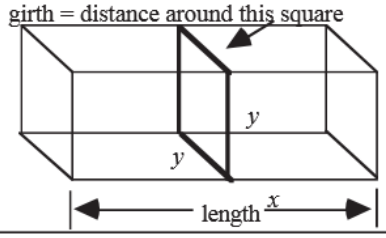
5. A fisheries biologist is stocking fish in a lake. She knows that when there are n fish per unit of water, the average weight of each fish will be $W(n) = 500 - 2n$, measured in grams. What is the value of n that will maximize the total fish weight after one season. *Hint: Total Weight = number of fish • average weight of a fish.*

$$W = n(500 - 2n)$$
$$W = 500n - 2n^2$$
$$0 = 500 - 4n$$
$$n = 125 \text{ fish}$$

6. The size of a population of bacteria introduced to a food grows according to the formula $P(t) = \frac{6000t}{60 + t^2}$ where t is measured in weeks. Determine when the bacteria will reach its maximum size. What is the maximum size of the population?

$$0 = \frac{(60 + t^2)(6000) - 6000t(2t)}{(60 + t^2)^2}$$
$$0 = 360000 - 6000t^2$$
$$t^2 = 60$$
$$t = \sqrt{60} \text{ - Week 8}$$
Size = 387 bacteria

7. The U.S. Postal Service will accept a box for domestic shipping only if the sum of the length and the girth (distance around) does not exceed 108 inches. Find the dimensions of the largest volume box with a square end that can be sent.



$$V = xy^2$$
$$V = (108 - 4y)y^2$$
$$V = 108y^2 - 4y^3$$
$$0 = 216y - 12y^2$$
$$0 = 12y(18 - y)$$
$y = 18$ in, $x = 36$ in

8. Blood pressure in a patient will drop by an amount $D(x)$ where $D(x) = 0.025x^2(30 - x)$ where x is the amount of drug injected in cm^3. Find the dosage that provides the greatest drop in blood pressure. What is the drop in blood pressure?

$$D(x) = 0.025x^2(30 - x)$$
$$D = .75x^2 - .025x^3$$
$$0 = 1.5x - .075x^2$$
$$0 = .075(20 - x)$$
$x = 20$ cm^3, Drop = 100 pts